

# p-ADIC AND ADELIC FREE RELATIVISTIC PARTICLE

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## Abstract

*We consider spectral problem for a free relativistic particle in p-adic and adelic quantum mechanics. In particular, we found p-adic and adelic eigenfunctions. Within adelic approach there exist quantum states that exhibit discrete structure of spacetime at the Planck scale.*

## 1. Introduction

Since 1987, there have been many interesting applications of p-adic numbers in various parts of theoretical and mathematical physics (for a review, see, e.g. Refs. 1-3). It is likely that the most attractive investigations have been in the Planck scale physics and non-archimedean structure of spacetime. One of the greatest achievements in that direction is a formulation of p-adic quantum mechanics<sup>4,5</sup> with its adelic generalization<sup>6</sup>. Adelic quantum mechanics unifies ordinary and p-adic ones, for all primes  $p$ .

There is an interplay of physical and mathematical reasons to investigate possible role of p-adic numbers and adeles in physics. Namely, all numerical results of experiments belong to the field of rational numbers  $\mathbb{Q}$ , which is dense in the field of real numbers  $\mathbb{R}$  and p-adic ones  $\mathbb{Q}_p$  ( $p$  is a prime number).  $\mathbb{R}$  and  $\mathbb{Q}_p$  ( $p = 2, 3, 5, \dots$ ) exhaust all possible numbers which can be obtained by completion of  $\mathbb{Q}$ . The set of adeles  $\mathbb{A}$  enables to regard real and p-adic numbers simultaneously. What is a limit in application of real numbers in description of spacetime? Do p-adic numbers play some role in physics? To answer these, and similar questions, one has to construct and examine p-adic and adelic models of quantum and relativistic physical systems.

Models at the Planck scale are of particular interest. In fact, if the Planck length  $l_0 = (\hbar G/c^3)^{1/2} \sim 10^{-33}$  cm is the elementary one then any other length  $x$  should be an integer multiple of  $l_0$ . So, if one takes  $l_0 = 1$  then one has  $x = n$ . Real distance is  $d_\infty(x, 0) = |n|_\infty = n$ , while the p-adic one is  $d_p(x, 0) = |n|_p \leq 1$ . Thus, p-adic geometry has to emerge approaching the Planck scale physics.

So far, different p-adic and adelic quantum models have been studied (see, e.g. Refs. 7-9).

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In this letter we examine p-adic and adelic quantum properties of a free relativistic particle. As a result of adelic approach we find some discreteness of space and time at the Planck scale.

## 2. Some p-Adic and Adelic Mathematics

Since majority of physicists are still unfamiliar with p-adic numbers and adeles we give here some basic facts about these attractive parts of modern mathematics (for a profound knowledge, see, e.g. Refs. 2, 10-12).

Any p-adic number  $x \in \mathbb{Q}_p$  can be presented in the unique way as an infinite expansion

$$x = x_{-k}p^{-k} + x_{-k+1}p^{-k+1} + \cdots + x_0 + x_1p + x_2p^2 + \cdots, \quad k \in \mathbb{N}, \quad (2.1)$$

where  $x_i = 0, 1, \dots, p-1$  are digits. p-Adic norm of a term in (2.1) is  $|x_i p^i|_p = p^{-i}$ . Since p-adic norm is the non-archimedean one, *i.e.*  $|u+v|_p \leq \max\{|u|_p, |v|_p\}$ , it follows that  $|x|_p = p^k$  in the representation (2.1).

There are mainly two types of analysis on  $\mathbb{Q}_p$ . The first one (i) is based on the mapping  $f : \mathbb{Q}_p \rightarrow \mathbb{Q}_p$ , and the second one (ii) is related to map  $\varphi : \mathbb{Q}_p \rightarrow \mathbb{C}$ , where  $\mathbb{C}$  is the set of complex numbers. We use both of these analysis in p-adic generalization of physical models: (i) in classical and (ii) in quantum mechanics. Derivatives of  $f(x)$  are defined as in the real case, but using p-adic norm instead of the usual absolute value function. For mapping  $\varphi(x)$  there is well-defined integration with the Haar measure. In particular, we use the Gauss integral<sup>2</sup>

$$\int_{|x|_p \leq p^\nu} \chi_p(\alpha x^2 + \beta x) dx = \begin{cases} p^\nu \Omega(p^\nu | \beta |_p), & | \alpha |_p \leq p^{-2\nu}, \\ \lambda_p(\alpha) |2\alpha|_p^{-1/2} \chi_p(-\frac{\beta^2}{4\alpha}) \Omega(p^{-\nu} | \frac{\beta}{2\alpha} |_p), & |4\alpha|_p > p^{-2\nu}. \end{cases} \quad (2.2)$$

$\chi_p(u) = \exp(2\pi i \{u\}_p)$  is a p-adic additive character, where  $\{u\}_p$  denotes the fractional part of  $u \in \mathbb{Q}_p$ .  $\lambda_p(\alpha)$  is an arithmetic complex-valued function<sup>2</sup> with the following basic properties:

$$\lambda_p(0) = 1, \lambda_p(a^2\alpha) = \lambda_p(\alpha), \lambda_p(\alpha)\lambda_p(\beta) = \lambda_p(\alpha+\beta)\lambda_p(\alpha^{-1}+\beta^{-1}), | \lambda_p(\alpha) |_p = 1. \quad (2.3)$$

$\Omega(|u|_p)$  is the characteristic function on  $\mathbb{Z}_p$ , *i.e.*

$$\Omega(|u|_p) = \begin{cases} 1, & |u|_p \leq 1, \\ 0, & |u|_p > 1, \end{cases} \quad (2.4)$$

where  $\mathbb{Z}_p = \{x \in \mathbb{Q}_p : |x|_p \leq 1\}$  is the ring of p-adic integers.

An adele  $a \in \mathbb{A}$  is an infinite sequence

$$a = (a_\infty, a_2, \dots, a_p, \dots), \quad (2.5)$$

where  $a_\infty \in \mathbb{R}$  and  $a_p \in \mathbb{Q}_p$  with the restriction that  $a_p \in \mathbb{Z}_p$  for all but a finite set  $S$  of primes  $p$ . The set of all adeles  $\mathbb{A}$  can be written in the form

$$\mathbb{A} = \bigcup_S \mathcal{A}(S), \quad \mathcal{A}(S) = \mathbb{R} \times \prod_{p \in S} \mathbb{Q}_p \times \prod_{p \notin S} \mathbb{Z}_p. \quad (2.6)$$

$\mathbb{A}$  is a topological space. It is a ring with respect to componentwise addition and multiplication. There is a natural generalization of analysis on  $\mathbb{R}$  and  $\mathbb{Q}_p$  to analysis on  $\mathbb{A}$ .

### 3. Free Relativistic Particle in p-Adic Quantum Mechanics

In the Vladimirov-Volovich formulation<sup>4</sup> (see also Ref. 5) one-dimensional p-adic quantum mechanics is a triple

$$\left( L_2(\mathbb{Q}_p), W_p(z), U_p(t) \right), \quad (3.1)$$

where  $L_2(\mathbb{Q}_p)$  is the Hilbert space of complex-valued functions of p-adic variables,  $W_p(z)$  is a unitary representation of the Heisenberg-Weyl group on  $L_2(\mathbb{Q}_p)$ , and  $U_p(t)$  is an evolution operator on  $L_2(\mathbb{Q}_p)$ .

$U_p(t)$  is an integral operator

$$U_p(t)\psi_p(x) = \int_{\mathbb{Q}_p} K_p(x, t; y, 0)\psi_p(y)dy \quad (3.2)$$

whose kernel is given by the Feynman path integral

$$K_p(x, t; y, 0) = \int \chi_p\left(-\frac{1}{h}S[q]\right)\mathcal{D}q = \int \chi_p\left(-\frac{1}{h}\int_0^t L(q, \dot{q})dt\right) \prod_t dq(t), \quad (3.3)$$

where  $h$  is the Planck constant. p-Adic Feynman path integral is investigated in Ref. 13, where it is shown that for quadratic classical actions  $\bar{S}(x, t; y, 0)$  the solution (3.3) becomes

$$K_p(x, t; y, 0) = \lambda_p\left(-\frac{1}{2h}\frac{\partial^2 \bar{S}}{\partial x \partial y}\right) \left| \frac{1}{h}\frac{\partial^2 \bar{S}}{\partial x \partial y} \right|_p^{1/2} \chi_p\left(-\frac{1}{h}\bar{S}(x, t; y, 0)\right). \quad (3.4)$$

Expression (3.4) has the same form as that one in ordinary quantum mechanics (*i.e.* with  $L_2(\mathbb{R})$ ). For a particular physical system, p-adic eigenfunctions are subject of the spectral problem

$$U_p(t)\psi_p^{(\alpha)}(x) = \chi_p(\alpha t)\psi_p^{(\alpha)}(x). \quad (3.5)$$

The usual action for a free relativistic particle<sup>14</sup>

$$S = -mc^2 \int_{\tau_1}^{\tau_2} d\tau \sqrt{\eta_{\mu\nu} \dot{x}^\mu \dot{x}^\nu} \quad (3.6)$$

is nonlinear and so unsuitable for quantum-mechanical investigations. However, a free relativistic particle can be treated as a system with the constraint<sup>15</sup>  $\eta_{\mu\nu}k^\mu k^\nu + m^2c^2 = k^2 + m^2c^2 = 0$ , which leads to the canonical Hamiltonian (with the Lagrange multiplier  $N$ )

$$H_c = N(k^2 + m^2c^2) , \quad (3.7)$$

and to the Lagrangian

$$L = \dot{x}_\mu k^\mu - H_c = \frac{\dot{x}^2}{4N} - m^2c^2N , \quad (3.8)$$

where  $\dot{x}_\mu = \partial H_c / \partial k^\mu = 2k_\mu$  and  $\dot{x}^2 = \dot{x}^\mu \dot{x}_\mu$ . Instead of (3.6), the corresponding action for quantum treatment of a free relativistic particle is

$$S = \int_{\tau_1}^{\tau_2} d\tau \left( \frac{\dot{x}^2}{4N} - m^2c^2N \right) . \quad (3.9)$$

From (3.9) it follows the classical trajectory

$$\bar{x}^\mu = \frac{x_2^\mu - x_1^\mu}{\tau_2 - \tau_1} \tau + \frac{x_1\tau_2 - x_2\tau_1}{\tau_2 - \tau_1} \quad (3.10)$$

and the classical action

$$\bar{S}(x_2, T; x_1, 0) = \frac{(x_2 - x_1)^2}{4T} - m^2c^2T , \quad (3.11)$$

where  $T = N(\tau_2 - \tau_1)$ .

All the above expressions from (3.7) to (3.11) are valid in the real case and according to p-adic analysis they have place in the p-adic one. Note that the classical action (3.11) can be presented in the form

$$\begin{aligned} \bar{S} = & \left[ -\frac{(x_2^0 - x_1^0)^2}{4T} - \frac{m^2c^2T}{4} \right] + \left[ \frac{(x_2^1 - x_1^1)^2}{4T} - \frac{m^2c^2T}{4} \right] \\ & + \left[ \frac{(x_2^2 - x_1^2)^2}{4T} - \frac{m^2c^2T}{4} \right] + \left[ \frac{(x_2^3 - x_1^3)^2}{4T} - \frac{m^2c^2T}{4} \right] = \bar{S}^0 + \bar{S}^1 + \bar{S}^2 + \bar{S}^3 \end{aligned} \quad (3.12)$$

which is quadratic in  $x_2^\mu$  and  $x_1^\mu$  ( $\mu = 0, 1, 2, 3$ ).

Due to (3.4) and (3.12), the corresponding quantum-mechanical propagator may be written as product

$$K_p(x_2, T; x_1, 0) = \prod_{\mu=0}^3 K_p^{(\mu)}(x_2^\mu, T; x_1^\mu, 0) , \quad (3.13)$$

$$\begin{aligned} K_p^{(\mu)}(x_2^\mu, T; x_1^\mu, 0) = & \lambda_p((-1)^{\delta_0^\mu} 4hT) |2hT|_p^{-\frac{1}{2}} \\ & \times \chi_p \left[ -\frac{1}{h} (-1)^{\delta_0^\mu} \frac{(x_2^\mu - x_1^\mu)^2}{4T} + \frac{1}{h} \frac{m^2c^2T}{4} \right] , \end{aligned} \quad (3.14)$$

where  $\delta_0^\mu = 1$  if  $\mu = 0$  and 0 otherwise.

Among all possible eigenstates which satisfy eq. (3.5), function  $\Omega(|x|_p)$ , defined by (2.4), plays a central role in p-adic and adelic quantum mechanics. Therefore, let us first show existence of  $\Omega$ -eigenfunction for the above relativistic particle. In fact, we have now 1+3 dimensional problem and the corresponding integral equation is

$$\int_{Q_p^4} K_p(x, T; y, 0) \Omega(|y|_p) d^4 y = \Omega(|x|_p), \quad (\alpha = 0), \quad (3.15)$$

where  $|u|_p = \max_{0 \leq \mu \leq 3} \{|u^\mu|_p\}$  is p-adic norm of  $u \in \mathbb{Q}_p^4$ , and

$$K_p(x, T; y, 0) = \frac{\lambda_p^2(4hT)}{|2hT|_p^2} \chi_p \left( -\frac{(x-y)^2}{4hT} + \frac{m^2 c^2 T}{h} \right). \quad (3.16)$$

Eq. (3.15), rewritten in a more explicite form, reads

$$\begin{aligned} & \frac{\lambda_p^2(4hT)}{|2hT|_p^2} \chi_p \left( \frac{m^2 c^2 T}{h} - \frac{x^2}{4hT} \right) \int_{\mathbb{Z}_p} \chi_p \left( \frac{(y^0)^2}{4hT} - \frac{x^0 y^0}{2hT} \right) dy^0 \\ & \times \prod_{i=1}^3 \int_{\mathbb{Z}_p} \chi_p \left( -\frac{(y^i)^2}{4hT} + \frac{x^i y^i}{2hT} \right) dy^i = \Omega(|x|_p). \end{aligned} \quad (3.17)$$

Using lower part of the Gauss integral (2.2) to calculate integrals in (3.17) for each coordinate  $y^\mu$  ( $\mu = 0, \dots, 3$ ), we obtain

$$\chi_p \left( \frac{m^2 c^2 T}{h} \right) \prod_{\mu=0}^3 \Omega(|x^\mu|_p) = \Omega(|x|_p), \quad |hT|_p < 1. \quad (3.18)$$

Since  $\prod_{\mu=0}^3 \Omega(|x^\mu|_p) = \Omega(|x|_p)$  is an identity, an equivalent assertion to (3.18) is

$$\left| \frac{m^2 c^2 T}{h} \right|_p \leq 1, \quad |hT|_p < 1. \quad (3.19)$$

Applying also the upper part of (2.2) to (3.17), we have

$$\frac{\lambda_p^2(4hT)}{|2hT|_p^2} \chi_p \left( \frac{m^2 c^2 T}{h} - \frac{x^2}{4hT} \right) \prod_{\mu=0}^3 \Omega \left( \left| \frac{x^\mu}{2hT} \right|_p \right) = \Omega(|x|_p), \quad |4hT|_p \geq 1, \quad (3.20)$$

what is satisfied only for  $p \neq 2$ . Namely, (3.20) becomes an equality if conditions

$$\left| \frac{m^2 c^2 T}{h} \right|_p \leq 1, \quad |hT|_p = 1, \quad p \neq 2, \quad (3.21)$$

take place.

Thus, we obtained eigenstates

$$\psi_p(x, T) = \begin{cases} \Omega(|x|_p), & |\frac{m^2 c^2 T}{h}|_p \leq 1, \quad |hT|_p \leq 1, \quad p \neq 2, \\ \Omega(|x|_2), & |\frac{m^2 c^2 T}{h}|_2 \leq 1, \quad |hT|_2 < 1, \end{cases} \quad (3.22)$$

$$(3.23)$$

which are invariant under  $U_p(t)$  transformation.

We have also  $\Omega$ -function in eigenstates

$$\psi_p(x, T) = \chi_p\left(\frac{m^2 c^2}{h}T\right) \Omega(p^\nu |x|_p), \quad \nu \in \mathbb{Z}, \quad |hT|_p < p^{-2\nu}. \quad (3.24)$$

This can be shown in the way similar to the previous case with  $\psi_p(x, T) = \Omega(|x|_p)$ .

The eigenstates without  $\Omega$ -functions are as follows:

$$\psi_p(x, T) = \chi_p\left(\frac{m^2 c^2 + k^2}{h}T\right) \chi_p\left(-\frac{kx}{h}\right), \quad (3.25)$$

where  $k^2 = -k^0 k^0 + k^i k^i$ , and  $kx = -k^0 x^0 + k^i x^i$ . Note that  $(m^2 c^2 + k^2)T = H_c \tau$  (see (3.7) and (3.11)).

#### 4. Free Relativistic Particle in Adelic Quantum Mechanics

According to Ref. 6, the main ingredients of adelic quantum mechanics are: (i) the Hilbert space  $L_2(\mathbb{A})$  of complex-valued functions on the space of adeles  $\mathbb{A}$ , (ii) a unitary representation  $W(z)$  of the Heisenberg-Weyl group on  $L_2(\mathbb{A})$ , and (iii) a unitary representation of the evolution operator  $U(t)$  on  $L_2(\mathbb{A})$ . In a sense, ingredients of quantum mechanics on adeles are some products of the corresponding objects from ordinary and p-adic quantum mechanics.

So, the evolution operator is defined by

$$U(t)\psi(x) = \int_{\mathbb{A}} K(x, t; y, 0)\psi(y)dy, \quad (4.1)$$

where  $t \in \mathbb{A}$ ,  $x, y \in \mathbb{A}$ ,  $\psi \in L_2(\mathbb{A})$ , and

$$U(t)\psi(x) = U_\infty(t_\infty)\psi_\infty(x_\infty) \prod_p U_p(t_p)\psi_p(x_p). \quad (4.2)$$

The spectral problem is given by

$$U(t)\psi^{(\alpha)}(x) = \chi(\alpha t)\psi^{(\alpha)}(x), \quad \alpha \in \mathbb{A}, \quad (4.3)$$

where  $\chi(\alpha t) = \chi_\infty(\alpha_\infty t_\infty) \prod_p \chi_p(\alpha_p t_p)$  is the additive character on  $\mathbb{A}$ .

Convergence of products and adelic consistency imply some conditions on p-adic constituents of an adelic object. For instance, any eigenfunction in (4.3) has the form

$$\psi(x) = \psi_\infty(x_\infty) \prod_{p \in S} \psi_p(x_p) \prod_{p \notin S} \Omega(|x_p|_p), \quad (4.4)$$

where  $S$  is a finite set of primes  $p$ .  $\psi_\infty(x_\infty) \in L_2(\mathbb{R})$  and  $\psi_p(x_p)$ ,  $\Omega(|x_p|_p) \in L_2(\mathbb{Q}_p)$  are eigenfunctions of ordinary and p-adic counterparts, respectively.

Adelic kernel of  $U(t)$  for relativistic free particle is

$$K(x, T; y, 0) = K_\infty(x_\infty, T_\infty; y_\infty, 0) \prod_p K_p(x_p, T_p; y_p, 0), \quad (4.5)$$

where  $K_p(x_p, T_p; y_p, 0)$  is given by (3.16), and  $K_\infty(x_\infty, T_\infty; y_\infty, 0)$  is the real counterpart, which has the same form as  $K_p(x_p, T_p; y_p, 0)$ . The corresponding arithmetic function  $\lambda_\infty(\alpha)$  is defined by  $\lambda_\infty(\alpha) = (1 - i \operatorname{sgn} \alpha) / \sqrt{2}$  and satisfies the same basic properties as  $\lambda_p(\alpha)$  (see (2.3)).

In order to complete the adelic spectral theory for a free relativistic particle let us now turn to the corresponding problem in ordinary quantum mechanics in the form

$$\int_{\mathbb{R}} K_\infty(x, T_2; y, T_1) \psi_\infty(y, T_1) dy = \chi_\infty(\alpha T_2) \psi_\infty(x), \quad (4.6)$$

where  $\chi_\infty(a) = \exp(2\pi i a)$ ,  $\psi_\infty(x, T) = \chi_\infty(\alpha T) \psi_\infty(x)$  and

$$K_\infty(x, T_2; y, T_1) = \frac{\lambda_\infty^2(4h(T_2 - T_1))}{|2h(T_2 - T_1)|_\infty^2} \chi_\infty\left(-\frac{(x - y)^2}{4h(T_2 - T_1)} + \frac{m^2 c^2 (T_2 - T_1)}{h}\right). \quad (4.7)$$

(For simplicity, we omitted index  $\infty$  for arguments in (4.7)). Using the Gauss integral

$$\int_{\mathbb{R}} \chi_\infty(\alpha x^2 + \beta x) dx = \lambda_\infty(\alpha) |2\alpha|_\infty^{-1/2} \chi_\infty\left(-\frac{\beta^2}{4\alpha}\right), \quad \alpha \neq 0, \quad (4.8)$$

we find the solution of (4.6) in the form

$$\psi_\infty(x, T) = \chi_\infty\left(\frac{m^2 c^2 + k^2}{h} T\right) \chi_\infty\left(-\frac{kx}{h}\right), \quad (4.9)$$

where  $k^2 = k_\mu k^\mu$ ,  $kx = k_\mu x^\mu$ .

From the above investigation it follows adelic eigenfunction for a free relativistic particle:

$$\psi(x, T) = \chi_\infty\left(\frac{m^2 c^2 + k_\infty^2}{h} T_\infty - \frac{k_\infty x_\infty}{h}\right) \prod_{p \in S} \psi_p(x_p, T_p) \prod_{p \notin S} \Omega(|x_p|_p), \quad (4.10)$$

where  $\psi_p(x_p, T)$  are given by (3.24) and (3.25). Note that  $x \in \mathbb{A}^4$  in (4.10), *i.e.*

$$x = \begin{pmatrix} x^0 \\ \vdots \\ x^3 \end{pmatrix} = \begin{pmatrix} x_\infty^0, & x_2^0, & \dots, & x_p^0, & \dots \\ \vdots & \vdots & & \vdots & \\ x_\infty^3, & x_2^3, & \dots, & x_p^3, & \dots \end{pmatrix} = (x_\infty, x_2, \dots, x_p, \dots), \quad (4.11)$$

and  $k \in \mathbb{A}^4$ ,  $T \in \mathbb{A}$ . Any adelic wave function may be obtained as superposition of eigenfunctions (4.10) by summation over  $S$  and integration over four-momentum  $k$ .

## 5. Concluding Remarks

We shown that a free relativistic particle may be regarded as a subject not only of ordinary but also of p-adic and adelic quantum mechanics. It is an exactly soluble theoretical model. In particular, we found p-adic and adelic eigenfunctions.

In order to interpret adelic wave function for a free particle let us consider its norm, *i.e.*

$$|\psi(x, T)|_\infty^2 = \prod_{p \in S} |\psi_p(x_p, T_p)|_\infty^2 \prod_{p \notin S} \Omega(|x_p|_p), \quad (5.1)$$

where we used  $|\psi_\infty(x_\infty, T_\infty)|_\infty^2 = 1$  and  $\Omega^2(|x_p|_p) \equiv \Omega(|x_p|_p)$ . As follows, (5.1) does not depend on real counterpart.

Comparison between theoretical predictions and experimental numerical data may be done only on rational numbers. Hence, consider (5.1) in points  $x_\infty = x_2 = \dots = x_p = \dots = x \in \mathbb{Q}$ . Since

$$\prod_{p \in S} \Omega(p^\nu |x|_p) = \begin{cases} 1, & x \in p^\nu \mathbb{Z} \\ 0, & x \in \mathbb{Q} \setminus p^\nu \mathbb{Z}, \end{cases} \quad \prod_{p \notin S} \Omega(|x|_p) = \begin{cases} 1, & x \in \mathbb{Z} \\ 0, & x \in \mathbb{Q} \setminus \mathbb{Z}, \end{cases} \quad (5.2)$$

it means that  $|\psi(x, T)|_\infty^2$  may be different from zero only in a finite number of rational points which are not integers. Extending standard interpretation of ordinary wave function to the adelic one, it follows that the probability of finding the particle in integer points is dominant.

There is a special (vacuum) state ( $S = \emptyset$ ) when all p-adic states are  $\Omega(|x_p|_p)$  and then particle can exist only in integer points of space and time. In ordinary quantum mechanics we label coordinates of space and time by real numbers which make continuum. However, using adeles to label space and time we obtain their discreteness in a natural way. Conditions  $|m^2 c^2 T/h|_p \leq 1$  and  $|hT|_p \leq |2|_p$ , which follow from (3.22) and (3.23), can be realized choosing  $h = c = m = 1$  as a system of units. If  $m$  is the Planck mass then spacetime contains the Planck length as the elementary one. Since  $|T|_p = |N\tau|_p \leq |2|_p$  and  $N$  is an arbitrary parameter, one can take  $N = 2$  and obtain  $|\tau|_p \leq 1$  for every  $p$ . Thus, the invariant intervals  $\tau$  as well as spacetime coordinates  $x^\mu$  are discrete. Let us notice that there exist discrete subgroups of the Poincaré group that transform discrete spacetime (lattice) into itself (see, e.g. Ref. 16). It is reasonable to expect that spacetime



discreteness is more manifest in quantum gravity models and we have such situation in adelic approach to quantum cosmology<sup>7,17</sup>.

Note that the above obtained results can be easily generalized to any number of spacetime dimensions.

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